CIMPA-BURKINA FASO School 2009

INDEX THEORY AND INTERACTIONS WITH PHYSICS

Abstracts of the mini-courses

Introduction to K-theory: Hervé Oyono (Clermont-Ferrand)

The aim of this course is to provide prerequisites for the K-theory of operator algebras used in the different lectures on index theory. It will start with a brief introduction to the spectral theory of C*-algebras. I will then give definitions and basic properties of K-theory and will focus on the following two issues:

- exact sequences in K-theory
- Bott periodicity.

I will end the course with a presentation on some examples of techiques used to compute K-theory groups.

K-theory and some applications: Adéremi Kuku (Univ. Iowa,USA)/(Nigéria)

I will concentrate on index theory while covering the following topics.

1) Grothendieck Group of exact categories with copious examples-- finitely generated projective modules over rings including group rings, orders, involutive Banach algebras, C*-algebras etc; topological vector bundles,,etc. Swan-Serre theorem; Some applications.
2) Some essentials of topological K-theory, including Bott periodicity theorem. K-theory for C*-algebras as a Z/2-graded algebra, and as topological K-theory, K-theory as a generalised cohomology theory, K-homology.Chern characters. Atiya-Singer index theorem.
3) Hilbert C*-modules and Kasparov K-K theory. Assembly maps and Baum-Connes conjecture --applications and ramifications.

The Atiyah-Singer index theorem: Moulay Benameur (Metz)

In this course I will explain the index theorem in K-theory in various geometric situations. I will focus on specific setups for which the Murray-von Neumann theory applies and provides interesting results. This course is meant for newcomers in the area as well as for students.

Index theorems and anomalies: Denis Perrot (Lyon)

The aim of this course is to show certain fundamental relations relating quantum field theory and noncomutative geometry. More specifically, we will study chiral anomalies in gauge theory from the point of view of noncomutative index theory. These two topics can indeed be seen as equivalent from a certain point of view. Chiral anomalies are a manifestation of the impossibility to preserve a classical symmetry on a quantum level, because of problems related to renormalisation. We will explain how this phenomenon can be interpreted as a *local* formula in the spirit of Connes and Moscovici, thereby introducing a few basic notions of cyclic cohomology.

Groupoïds and index theory for stratified spaces: Jean-Marie Lescure (Clermont-Ferrand)

The aim of this course is to explain how introducing groupoids into the picture can lead to a generalisation of the Atiyah-Singer index theorem to stratified spaces.
We will provide an introduction to the concepts of groupoids, stratified spaces and recall some of the basic aspects of Kasparov's bivariant K-theory.
We will first state and then reformulate Atiyah and Singer's classical theorem for closed manifold in terms of these concepts and show how this reformulation allows for singularities, thereby leading to the announced generalisation of the index theorem.
Finally, we will mention other uses of groupoids in the study of stratified spaces, in particular Poincaré duality and wrong way functoriality in K-theory.

A few analytic tools underlying the analytic index theorem: Sylvie Paycha (Clermont-Ferrand)

We shall first recall a few geometric concepts such as Clifford connection, curvature and Chern-Weil forms. We shall then show the Bochner-Lichnerowicz formula which lies at the heart of the analytic index theorem. We shall moreover recall certain analytic notions such as differential operators which will be illustrated by Dirac type operators or generalised Laplacians, complemented by properties of elliptic operators on closed manifolds. We shall provide various ways of expressing the index of an operator which involve different types of traces, while insisting on the locality of the index formula which involves Chern-Weil forms.