CIMPA-UNESCO-EGYPT School
Recent Developments in the Theory of Elliptic Partial Differential Equations

Arab Academy for Science and Technology, Alexandria, Egypt
from Monday, January 26 to Tuesday, February 3, 2009

Abstracts of courses and seminars

M.J. Esteban: Introduction to variational methods for non-compact problems

Minimization, mountain pass, saddle points problems, etc., are more difficult to study when a group of non-compact invariances is present in the problem. The concentration-compactness method allows one to treat this kind of situations. In this course, this method will be presented on several non-compact variational problems arising in Physics.

V. Ferone: Symmetrization methods and applications to PDEs

The aim of the course is to show how symmetrization techniques can be used to obtain informations about the solutions to elliptic and parabolic problems. By symmetrization techniques we mean all those tools (isoperimetric inequalities, rearrangements, etc.) which give the possibility to compare, and then to estimate, the solution to a given problem with the solution to a problem of the same type, but having some symmetries. In the first part of the course the definition of the rearrangement of a function will be given and the main properties of rearrangements will be discussed. In the second part it will be shown on model cases how symmetrization techniques can be used to get comparison results. The use of such results in order to get regularity and existence results will be also illustrated.

F. Flori: Properties of some partial differential equations related to free boundary problems

In this presentation, we focus on some questions arising in the resolution of partial differential equations defined on free boundary domains. We illustrate our talk with some problems occurring in fluid mechanics. We also give numerical examples connecting modelling aspects with theoretical aspects.

M. Jaoua: Data completion and applications to some inverse problems

In this course we shall present algorithms of identification for inverse problems, which are well known to be ill-posed, starting from incomplete boundary measurements. This requires to deal with the data completion issue, which leads to a Cauchy problem for an elliptic operator, an ill-posed problem also. Suitable methods of regularization are needed in order to remove undesirable behaviors. Applications of the course will include identification of cracks and rebuilding of impedances which identify the location of corrosion zones.

The plan of the course will be as follows:
- Cauchy problem: ill-posedness of the problem, definition of compatible data;
- Energy functionals for error computation: auto-regularizing character, robustness and minimization algorithms;
- Iterative regularization method of Tikhnonov;
- Applications: detection of cracks and identification of electric impedances.

F. Murat: Non elliptic problems with right-hand side in L^1

In these seminars, I will consider the problem $-\mathrm{div}\, a(x, Du) = f$ in Ω, $u = 0$ on $\partial\Omega$, when the operator $-\mathrm{div}\, a(x, Du)$ is a monotone coercive operator acting on $W_0^{1,p}(\Omega)$ (with $1 < p \leq N$) and when f belongs to $L^1(\Omega)$.

The main difficulty of the problem is to define a convenient notion of solution. I will introduce the notion of renormalized solution and prove that the problem has a renormalized solution which is unique and which depends continuously on f.

F. Pacard : Geometric aspects of the Allen-Cahn equation

The first talk will be devoted to the existence of a new type of entire solutions for the Allen-Cahn equation, which appears in the modeling of phase transition problems. The solutions we are interested in have the property that their nodal sets are asymptotic to k-half lines. I will also explain the counterpart of these solutions for semilinear elliptic equation which appear in the study of stationary solutions for the nonlinear Schoedinger equation.

The second talk will be devoted to the study of the moduli space of the solutions described in the first talk. In particular, we will explain how to determine the dimension of the corresponding moduli spaces in terms of k, the number of nodal curves.

A. Porretta : Elliptic equations with first order terms

We present the main features of some basic models of second order elliptic equations with first order terms. The two main examples are elliptic equations with transport terms or second order Hamilton-Jacobi equations which may arise, for instance, from stochastic control problems. This kind of problems are non coercive but (at least formally) enjoy the maximum principle. Both notions of weak solution and viscosity solution have been proved to be useful in such context. In the first part of the course we present different issues concerning a priori estimates, compactness, uniqueness of solutions. In the second part we focus on some significant examples and we discuss the problem of gradient bounds in connection with singular limits like the vanishing viscosity limit or the ergodic limit.

D. Smets: Geometric flows and motion of thin interfaces

The plan of the course will be as follows:
– Some examples of wall energies in physics of materials;
– Singular limits of non convex energies of Modica-Mortola type;
– Definitions of weak notions of the motion by mean curvature;
– Application to the dynamics of the motion of interfaces.

M. Vanninathan: Homogenization problems and Bloch waves

It is not an exaggeration to say that the publication in 1978 of the book *Asymptotic analysis for periodic structures*, by A. Bensoussan, J.-L. Lions & G. Papanicolaou resulted in a blow-up of research activities in homogenization problems. Several methods were proposed. Roughly they can be described as follows: Physical space methods, Fourier space methods and Phase space methods. In these lectures, I will focus on a class of problems for which Fourier techniques are applicable and highlight the advantages of this approach by re-establishing some old results (with important improvements) as well as by obtaining some new results (e.g. the negative dispersion in heterogeneous media). This approach enabled us to formulate sufficient conditions for homogenization on the Fourier side. We also treat models where these structural conditions are violated. Both scalar equations and systems of equations will be considered.