Résumé de cours

Aline Bonami

SOME APPLICATIONS OF CALDERON-ZYGMUND THEORY TO HANKEL AND TOEPLITZ OPERATORS

Hankel and Toeplitz matrices play an important role in many applications of mathematics. In the infinite dimensional case, the properties of the corresponding operators rely on those of their symbol. We will start with complex analysis in the unit disc to explain the classical theory of Hankel and Toeplitz operators, then give some more recent developments and emphasize their connections with Calderon-Zygmund theory: in particular commutators of singular integrals and bilinear Hilbert transforms.

Oscar Blasco

BILINEAR HILBERT TRANSFORM AND OTHER MULTIPLIERS. TRANSFERENCE AND APPLICATIONS

The Hilbert transform in the real line is a bounded operator on certain Lebesgue spaces, and is an example of a very important multiplier in Analysis. The bilinear Hilbert transform and other bilinear multipliers have been subject of study since Calderón's conjecture was proved. Boundedness results for several bilinear multipliers will be presented. Also, by using commutators, a relationship between linear and bilinear operators will be established.
Different methods to transfer their bounded ness to other groups and other contexts will be analyzed, generalizing the linear De Leew's methods as well as the transference methods of Coiffman-Weiss. By this procedure, applications to ergodic theory will be obtained.

Luis Caffarelli

NON LINEAR PROBLEMS INVOLVING FRACTIONAL POWERS OF THE LAPLACIAN

We will discuss several problems from the calculus of variations and fluid dynamics involving fractional Dirichlet integrals and fractional powers of the Laplacian: Optimal regularity of constrained variational problems, harmonic extensions of fractional dimensions, and the regularity of solutions to the quasi geostrophic equation.

Jacek Dziubanski

HARMONIC ANALYSIS ASSOCIATED WITH CERTAIN SEMIGROUPS OF LINEAR OPERATORS

Classical function spaces on R^d like Hardy spaces, Sobolev spaces, Triebel-Lizorkin spaces, could be very often defined and studied by means of heat or Poisson semigroups and functional calculi associated with the Laplace operator. The purpose of the lectures will be introducing some new function spaces that are related to other operators and semigroups they generate. An example of such operators could be a.g. the harmonic oscillators or the Bessel operator. We shall discuss

several properties of these function spaces and bounded ness of relevant operators on them, like spectral multiplier operators, Riesz transforms, etc.

Alberto Grünbaum

APPROXIMATION IN MEDICAL TOMOGRAPHY. SHANNON'S PROBLEM IN INFORMATION THEORY AND THE ORIGIN OF THE BISPECTRAL PROBLEM.

The typical problem in tomography consists in obtaining a good approximation of a function of several variables starting from incomplete and inexact data on some of its one-dimensional projections. In the case of limited angle tomography this gives rise to a version of the classic problem of Shannon: what "useful" information is contained in a band of frequencies of a function with compact support?

The detailed analysis of that problem eventually leads to consider integral operators that (miraculously) commute with a differential operator. This, in turn, (and for even more mysterious reasons) conducts to certain non-linear equations associated to the names of Korteweg-de Vries, Toda, Virasoro, etc. that describe the solutions of the bispectral problem.

Carlos Pérez

APPLICATIONS OF REAL AND COMPLEX ANALYSIS TO COMMUTATORS OF B.M.O. FUNCTIONS WITH SINGULAR INTEGRALS AND GENERAL LINEAR OPERATORS

In this course we will introduce some techniques from Harmonic Analysis that will lead to the study of Commutators of Calderón-Zygmund Singular Integrals with B.M.O functions. A first goal will be to present some classical techniques such as "good-lambda" inequalities, Calderón-Zygmund theory and some basics from the Ap theory of weights. Then we will prove the classical strong type estimates due to Coifman-Rochberg-Weiss for these type of commutators. We will show a different proof, using some ideas from complex Analysis, for general linear operators with the property that they are bounded on L2 with A2 weights, not just Singular integrals. Then we will show more delicate properties of these Commutators, in the case of Singular Integrals, by means of the Real Analysis techniques previously explained. In particular we will show the intimate connection with iterations of the Hardy-Littlewood maximal operator. If there is time we will show either some relationship with the extrapolation theory of weights or we will outline some recent results in the context of multilinear Harmonic Analysis. Finally we will present some problems in the area.

Sundaram Thangavelu

HARMONIC ANALYSIS ON HEISENBERG NILMANIFOLDS

In these lectures we plan to do some harmonic analysis on nilmanifolds of the *form* $M = H^n_{\Gamma}$ where H^n is the $(2n+1)$-dimensional Heisenberg group and Γ a discrete co-compact subgroup. The main objective is to get an explicit decomposition of the right regular representation of H^n on $L^2(M)$ into irreducible subrepresentations. This will be achieved by means of Weil-Brezin transforms and the Stone-von Neumann theorem. We plan to investigate translation invariant subspaces of $L^2(M)$ the spectrum of the sublaplacian, heat equation, Segal-Bargmann transform, theta functions and Hardy spaces associated to nilmanifolds.

José Luis Torrea

VECTOR VALUED TRANSFERENCE

We shall describe the transference method introduced by Cotlar (1955) and Calderón (1968). The power of the method was established in the comprehensive survey written by R. Coiffman and G. Weiss (1977). Since then several results involving the transfer of the boundedness of maximal operators and square functions associated to family of operators have been obtained. The aim of this course is to develop the method in a vector-valued setting. This generalization, whose proof is a straightforward modification of the proof of the original results, allows to get new results in different fronts of Harmonic Analysis. Special attention shall be given to dimension free results. A short introduction to vector-valued Fourier Analysis will be also given.