CIMPA-UNESCO-ARMENIA SCHOOL

Nonlinear analysis and Geometric PDE

15-24 June 2008, Tsaghkadzor, Armenia.

Abstracts of talks

Luis A. Caffarelli

The porous media equation and geometric methods in non linear PDE

In this course we will present a (relatively) simple free boundary problem, the boundary of the support of the porous media equation, and use it to illustrate basic methods:

The role of renormalization to obtain basic estimates

How regularity in space implies regularity in time for diffusion equations

The role of particular solutions to guide us in our problem and as technical tools

The classification of global solutions as a tool for regularity of the free boundary

Henri Berestycki

"Reaction-diffusion equations and propagation phenomena"

Reaction diffusion equations and systems play a central role in reactive flows, chemical waves, combustion theory, systems undergoing phase changes and in modelling a variety of biological and ecological systems, in particular in describing different kinds of invasions. The important features of these equations are the propagation of fronts and the spreading or quenching of confined initial data. In this series of lectures, I will first review the classical results in the framework of the classical homogeneous reaction-diffusion equations. I will describe some of the models and recall the basic results of Kolmogorov, Petrovsky and Piskunov, Aronson and Weinberger and of Fife and McLeod. The main thrust of the course will then be to present the most recent developments regarding non homogeneous media. First, I will show several cases where there still are travelling waves but which are non planar. Then, I will discuss in detail the case of periodic media. There, the notion of pulsating travelling wave extends that of travelling wave. Also, one can derive formulas for the asymptotic speed of spreading. I will present properties dealing with the qualitative influence of various factors on the speed of propagation. Lastly, I will present some results on propagation and generalized fronts in very general non homogeneous media. These lectures build on several recent joint works with Francois Hamel, Nikolai Nadirashvili and Hiroshi Matano.

Yann Brenier

Optimal transport, convection and magnetic relaxation

We first establish a connection between Optimal Transport Theory and classical Convection Theory for geophysical flows. Our starting point is the model designed few years ago by Angenent, Haker and Tannenbaum to solve some Optimal Transport problems by a gradient flow approach. We interpret this geometric equation as a generalization of the Darcy-Boussinesq equations for use in Convection Theory. This suggests that the Navier-Stokes-Boussinesq equations, the basic model in Convection Theory, provide a good framework for Optimal Transport related problems, such as Hoskins' Semigeostrophic equations and some fully nonlinear version of some Chemotaxis equations. In a different direction, we introduce a "stringy" generalization of the Angenent Haker Tannenbaum model. This model is closely related to the Magnetic Relaxation model investigated by Arnold and Moffatt. It is a gradient flow for which the equilibrium states are just the (variational) solutions of the Euler equations for incompressible flows.

Stefan Luckhaus

Minimal surfaces, free boundary problems and geometric measure theory

The basic regularity theory for minimal surfaces according to E. DeGiorgi is reviewed and the general varifold theory is introduced. The rectifiability theorem of Allard for varifolds of bounded mean curvature is proved. Here the aim is to work with explicit estimates as far as possible. The applications of these geometric measure theory results are existence proofs for Stefan-Gibbs-Thompson free boundary problems, modelling change of phase with surface tension effects. The literature are some recent papers by the author and Leon Simon's book "Geometric measure theory".

Nina N. Uraltseva

Regularity of free boundaries in Obstacle-type problems

This lectures (4 times 1h) will be devoted to questions of regularity of free boundaries, in connection with the obstacle-type problems. The phrase "Obstacle" can sometimes be misleading, since in some of the problems we do not have any obstacles. In general we treat problems where there is a qualitative difference between two adjacent phases of the solution of an elliptic equation. The only part, reminiscent of the obstacle problem, is the equation itself.

We consider solutions of equations of the type

$$\Delta u = f(x,u,\nabla u) \quad \text{in} \quad D, \qquad (*)$$

where D is an open set in $\R^n$ and the right hand side term is supposed to be piecewise continuous, having jumps at some values of the arguments u and ∇u. We also suppose that there is a certain apriori unknown subset $\Omega=\Omega(u)$ of D where the corresponding equation (*) is "good" and we are interested in the regularity of the free boundary $\Gamma (u)= \partial \Omega \cap D$.

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