

CIMPA -UNESCO-MOROCCO SCHOOL

Riemannian Geometry, pseudo-Riemannian Geometry and  
Mathematical Physics

Marrakech May 19 - 30, 2008

Abstracts

**Abdelhak Abouqateb (Université Cadi-Ayyad, Marrakech) and  
Aziz El Kacimi (Université de Valenciennes):**

Title: *Principal bundles and connections*

Abstract: In this course, we will introduce the basic material on principal bundles and connections in view of its applications in mathematical physics.

**Helga Baum (Humboldt-Universität Berlin):**

Title: *Introduction to holonomy theory of Riemannian and Lorentzian manifolds*

Abstract: In this course, we will introduce basic material on holonomy groups of (pseudo)-Riemannian manifolds. After presenting the long known classification results in the Riemannian case, we will focus on recently obtained classification results for Lorentzian holonomy groups and discuss local and global Lorentzian metrics with special holonomy. We will also explain the relation between holonomy groups and special spinor fields.

**François Béguin (University Paris VI, France) and Abdelghani  
Zeghib (ENS Lyon, France):**

Title: *Constant curvature spacetimes*

Abstract: The study of constant curvature spacetimes allows one to introduce certain notions and problems of Global Lorentzian Geometry in a quite simplified context. Indeed, constant curvature spacetimes may exhibit a very rich topology and global geometry, but have — by definition — a trivial local geometry.

During this course, we shall present some classical concepts of General Relativity (e.g. *past* and *future* of a set, *causal spacetime*, *globally hyperbolic spacetime*, *time function*, *conformal boundary of a spacetime*, *initial singularity of a spacetime*, *horizon of a black hole*) and illustrate these concepts in the context of constant curvature spacetimes. Simultaneously, we will explain how the notions of *developing map* and *holonomy group* allow one to give a complete description of large classes of constant curvature spacetimes. A certain number of open questions arise from this description.

This course — which we hope to be interesting by itself — can also be considered as an introduction to the course on *Bianchi cosmological models*.

**François Béguin (University Paris VI, France) and Abdelghani Zeghib (ENS Lyon, France):**

Title: *Bianchi Cosmological models*

Abstract: Bianchi cosmological models are *spatially homogenous spacetimes*. More precisely, they are manifolds of the form  $M = I \times G$  where  $I \subset \mathbb{R}$  is an interval and  $G$  is a Lie group, endowed with a lorentzian metric of the form  $-dt^2 + g_t$  where  $(g_t)_{t \in I}$  is a family of left-invariant Riemannian metrics on  $G$ .

The physical content of a spacetime  $M$  is encoded by a non-linear PDE on its lorentz metric: the so-called Einstein equation. The first goal of our course will be to explain why, in the context of Bianchi cosmological models, this PDE reduces to a second order *ordinary differential equation* on the family of metrics  $(g_t)_{t \in I}$ . Then we will show that, for certain Lie groups  $G$ , this equation has a rather simple dynamics, whereas in some other cases, it exhibits a very rich and chaotic dynamics (which is still not fully understood).

We think that the main interest of the course on Bianchi Cosmological Models is that it stands at the crossroad of a wide variety of mathematical and physical fields : Cosmology, Global Riemannian and Lorentzian Geometry, PDE's, Dynamical Systems Theory, Lie Groups Theory,...

**Mohamed Boucetta (Université Cadi-Ayyad, Marrakech):**

Title: *Curvatures in Riemannian and pseudo-Riemannian geometry*

Abstract: We introduce the different notions of curvature associated with a pseudo-Riemannian metric and give some of their important properties. We also give some classical results on the interactions between curvature and topology.

**Piotr Chrusciel (Université François Rabelais, Tours, et University of Oxford):**

Title: *Introduction to black holes*

Abstract: In this mini-course we will introduce and briefly describe the Schwarzschild, Kerr, and Emparan-Reall black holes. We will present the key theorems of black hole theory: topological censorship, area, and topology theorems. We will prove uniqueness of static vacuum black holes.

**Ahmad El Soufi (Université François Rabelais, Tours)**

Title: *Natural operators in Riemannian Geometry*

Abstract: The first aim of this course is to give some basic material about various Laplace and Schrödinger type operators associated with various Riemannian settings such as Laplace-Beltrami operator, Lichnerowicz Laplacian, Jacobi operator associated with stability problems in minimal surfaces theory, isoperimetric problem or motion by mean curvature. Relationships between eigenvalues and (intrinsic or extrinsic) curvature will be investigated, as well as applications concerning extremal properties of round spheres. This course and Harrell's course are complementary.

**Evans Harrell (Georgia Institute of Technology, Atlanta)**

Title: *Mathematics of quantum mechanics on thin structures.*

Abstract: Microelectronic devices known as quantum wires and waveguides are in production on scales where the electrons are subject to quantum effects and described with the Schrödinger equation. These devices are macroscopic in one or two dimensions but on the scale of nanometers in at least one dimension.

The course will begin a discussion of some current research in nanophysics laboratories and the mathematical models used in the field. The idealization of a thin domain in quantum mechanics as a restriction to a submanifold is one of the more important models, so careful asymptotics will be carried out for this model. The geometry of the submanifold will contribute explicitly to an effective potential in the limiting Schrödinger equation.

Spectral theory of elliptic PDEs will be reviewed and applied to Schrödinger operators on manifolds. The effect of the geometry on the eigenvalues (energy levels) will be investigated with a variety of techniques, some variational and some purely algebraic, including "sum rules" for

the spectra and "universal bounds" on eigenvalues. Special features of the Schrödinger operators of quantum wires and waveguides will be considered from physical, geometric, and analytical points of view. Connections will be made to isoperimetric theorems, Weyl asymptotics (asymptotics for high energies), and semiclassical approximation. The latest known results of these types will be explained.

**Aziz Ikemakhen (Université Cadi-Ayyad, Marrakech)**

Title: *Introduction to spinorial geometry*

Abstract: In this courses, we introduce Cliffords algebras and their representations, the spin groups and their representations. We introduce the notion of spin structure and spinc structures on a manifolds and constructs the spinorial bundles and their canonical connections.

**Jim Isenberg (University of Oregon)**

Title: *Finding Solutions of the Einstein Constraint Equations*

Abstract: The initial value approach is the most widely used method for finding and studying solutions of the Einstein gravitational field equations, and the first step in carrying out an initial value construction is to solve the Einstein constraint equations. In this set of lectures, we discuss some of the methods which have been developed for studying the constraint equations and their solutions. We particularly focus on the conformal and conformal thin sandwich methods, and on gluing techniques.

**Frank Pacard (University Paris XII)**

Title: *TBA*