

Number Theory and Automorphic Forms

Organizers:

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Place: IPM, Tehran, Iran

CONFERENCE PROPOSAL SUMMARY

We propose a week long conference on Analytic Number Theory, Arithmetic Geometry and Langlands Program on Automorphic Forms to be held at the Institute for Studies in Theoretical Physics and Mathematics (IPM). The conference will be preceded by several preparatory courses. Several young Iranian mathematicians residing abroad will also visit during the spring, summer and fall of 2006 for this purpose.

RATIONALE

For the past two decades the Iranian mathematical community has been quite successful in training young talented students, and many of them have become internationally recognized research mathematicians. Number theory, automorphic forms and arithmetic geometry have attracted a number of young mathematicians in Iran. While there have been many national conferences in Iran in which sporadic lectures have been given by a few number theorists, there has never been any international conference organized in number theory anywhere in the country. It is generally felt that an international conference on Number Theory and Automorphic Forms preceded by preparatory courses given in the semesters before the conference will have a lasting impact on the development of mathematics in Iran. In fact, this is expected to strengthen the interests of young talented students, as well as lay the foundations for better relations between Iranian mathematical community and the leading experts in number theory and related topics in other countries. It is for these reasons that IPM, which is the

leading research institute in theoretical physics and mathematics in Iran, has decided to have a conference in number theory over the period of December 16-22, 2006 in Tehran. IPM is very experienced in running international conferences. It is located in a pleasant environment on the foothills of Alborz mountains north east of Tehran and is quite appropriate for running a meeting of such significance and size.

DESCRIPTION OF THE CONFERENCE

To have a balanced program, the organizers have invited experts in analytic number theory, arithmetic geometry and automorphic forms from a number of different countries. This is particularly appropriate since in the recent years each of these subjects has seen significant progress. For example, recently there has been striking progress in establishing Langlands functoriality and the Fontaine-Mazur conjectures. Notable among these are Lafforgue's proof Langlands correspondence for $GL(n)$ over function fields, the work of Harris-Taylor and Henniart on establishing the same result over local fields, the work of Cogdell{Kim{Piatetski{Shapiro{Shahidi on establishing different instances of functoriality, including the 3rd and the 4th symmetric powers of forms on $GL(2)$, and finally the proof of the fundamental lemma for unitary groups by Laumon and Ngo. The latter is a crucial ingredient in the use of Arthur's trace formula in establishing functoriality for endoscopic cases. When fundamental lemmas of different kinds are established, the stabilization of the trace formula, recently settled by Arthur, should lead to many important cases of functoriality. One significant consequence of the functoriality for Sym^3 and Sym^4 of automorphic forms are surprising improvements on the bound towards the Ramanujan and Selberg conjectures by Kim, Sarnak and Shahidi. A remarkable consequence of stabilization of the trace formula will be an eventual proof of the Hasse-Weil conjecture for the zeta functions of Shimura varieties. This problem has attracted the attention of a number of leading experts in the field. Beside the results towards the Selberg conjecture discussed above, remarkable progress has also been made on the analytic theory of L-functions and their subconvexity, and particularly strong results have been established by Bernstein-Reznikov, Sarnak and Venkatesh. Connections with ergodic theory have been also thoroughly studied by Lindenstrauss, Rudnick-Sarnak and Watson with surprising results. The progress on the arithmetic aspects has been equally impressive. With Wiles' celebrated proof of Fermat's Last Theorem through that of Shimura-Taniyama conjecture in the semistable case which is now fully settled for curves over the rationals, doors were opened to the use deformation theory and its consequences in automorphic forms. This has allowed Taylor and his collaborators to prove many new cases of Artin conjecture for odd icosahedral Galois representations,

and the Sato-Tate conjecture. At the same time Skinner and Urban have recently proved the main conjecture of Iwasawa theory for elliptic curves over cyclotomic \mathbb{Z}_p -extensions of rationals for ordinary primes. Finally, one should mention a host of other results in connection with both p -adic modular forms and Iwasawa theory, notably the work of Coates and his collaborators, Bouganis-Dokchitser, Darmon-Tian, Kato and Zhang. Recently there has been very significant progress towards the Fontaine-Mazur conjecture (Breuil, Kissin).

LECTURES (tentative)

As it is evident from the list of confirmed invited speakers, we are in the position of covering many areas of recent progress discussed above. Although it is still early to have an official schedule, we tentatively hope to have lectures by Arthur, Clozel, Labesse and Waldspurger on different aspects of the trace formula and its applications. Asgari, Cogdell, Shahidi and Sarnak are expected to discuss the recent progress in functoriality over number fields and analytic aspects of L -function. Many other aspects of automorphic forms, representation theory and number theory will be covered through the talks by Harris, Kudla, Kutzko, Patterson, Takloo, Taylor and Vigneras. There will be a good number of arithmetic geometers present in the meeting who we expect to cover different aspects of the subject. We look particularly forward to have talks on different aspects of p -adic L -functions and recent progress in Iwasawa theory from Kassaei, Schneider, Skinner and Zhang. A decision on whether some of these speakers will give several lectures will be made later.

As explained earlier, the conference will be preceded by lectures and courses given by visitors and a number of younger Iranian number theorists. These lectures are currently in progress. Moreover, Shahshahani has taught an intensive two semester course on related subjects at Sharif University, where most talented mathematics students usually attend.

Preliminary list of invited speakers (not including organizers):

- A. Asgari (Oklahoma State University)
- L. Clozel (University of Paris, Orsay)
- J. Coates (Cambridge University)
- J. Cogdell (Ohio State University)
- G. Faltings_ (MPI, Bonn)
- M. Harris (University of Paris)
- G. Henniart_ (University of Paris, Orsay)
- H. Iwaniec (Rutgers University)

P. Kassaei (McGill University, University of London)
S. Kudla (University of Maryland)
P. Kutzko (University of Iowa)
J-P. Labesse (University of Marseille)
J-S. Li_ (University of Hong Kong)
P. Lochak (University of Paris)
Y. Nesterenko (Moscow State University)
S. Patterson (Göttingen)
D. Ramakrishnan_ (Caltech)
P. Schneider (University of Muenster)
R. Sharifi (McMaster University)
C. Skinner (University of Michigan)
R. Takloo (Princeton University)
R. Taylor (Harvard University)
M.-F. Vigneras (University of Paris)
S. Vostokov (St. Petersburg)
J.-L. Waldspurger (University of Paris)
S. Zhang (Columbia University)
W. Zudilin (Moscow State University)

Remark- All local expenses will be covered by IPM, and the invited speakers will reside at the IPM Guesthouse. Often visitors are interested in visits to historical sights. IPM staff is experienced in making such arrangements.