APPENDIX A
ABSTRACTS OF TALKS

1. **Prof. Shiro Goto** (*Meiji University, Japan*)

Title: *Cohen-Macaulayness in the associated graded rings of certain $\mathfrak{m}$-primary ideals in a Noetherian local ring*

Let A be a Noetherian local ring with the maximal ideal $\mathfrak{m}$. Let Q be a parameter ideal in A and let $I = Q : \mathfrak{m}^\ell$ with $\ell > 0$ an integer. The main problem of my lectures is the following. I shall give some answers in the case where $\ell = 1$ or 2. The problem is open, when $\ell \geq 3$, even in the case where A is a regular local ring.

Problem 1.
(1) *Is the parameter ideal Q a reduction of I?*
(2) *When this is the case, compute the reduction number*

$$\mathrm{r}_Q(I) = \min\{0 \leq n \in \mathbb{Z} \mid I^{n+1} = QI^n\}$$

of I with respect to Q.
(3) *Give the conditions on Q and A under which the Rees algebra $\mathrm{R}(I) = \bigoplus_{n\geq 0} I^n$, the associated graded ring $\mathrm{G}(I) = \bigoplus_{n\geq 0} I^n/I^{n+1}$, and the fiber cone $\mathrm{F}(I) = \bigoplus_{n\geq 0} I^n/\mathfrak{m}I^n$ of I are Cohe-Macaulay (Gorenstein, Buchsbaum) rings.*

In my lectures I will give some results about the problem in the case where $\ell \leq 2$ and provide some additional questions.

Lecture 1: Firstly I shall prove Trung–Ikeda Theorem (or Goto–Shimoda Theorem), which reduces Cohen-Macaulayness of $\mathrm{R}(I)$ to the one of $\mathrm{G}(I)$ and its a-invariant. Lipman's result on Cohen-Macaulayness of $\mathrm{R}(I)$ for ideals I in regular (or, pseudo-rational) local rings shall be discussed. I will summarize some conditions on $\mathfrak{m}$-primary ideals I, which implies Cohen-Macaulayness of $\mathrm{G}(I)$ and $\mathrm{F}(I)$.

Lecture 2: For Buchsbaumness in $\mathrm{R}(I)$ we still lack such a characterization or a theorem of Goto–Shimoda type. In certain special cases, one however has the result which asserts the equivalence of Buchsbaumness in $\mathrm{R}(I)$ and $\mathrm{G}(I)$, similarly as Cohen-Macaulayness in the theorem of Lipman. I will discuss the case where A is a regular local ring of dimension 2, or a little more generally, the case where A is a rational singularity of dimension 2.

Lecture 3. The case where $I = Q : \mathfrak{m}$ with A a Cohen-Macaulay local ring shall be studied. The result says that $I^2 \neq QI$ if and only if $\overline{Q} = Q$, that is Q is integrally closed in A, or equivalently A is a regular local ring and the A-module m$/Q$ is cyclic. Therefore, if A is a Cohen-Macaulay local ring which is not regular, for every parameter ideal Q in A, one has the equality $I^2 = QI$ where $I = Q : \mathfrak{m}$, so that both the rings $\mathrm{G}(I)$ and $\mathrm{F}(I)$ are Cohe-Macaulay, whence so is the Rees algebra $\mathrm{R}(I)$ of I, provided dim $A \geq 3$. The prrof partially depends on the theory of $\mathfrak{m}$-full ideals, that I shall briefly summarize in Lecture 3.

Lecture 4: The case where $I = Q : \mathfrak{m}^2$ in which A is a Gorenstein local ring shall be explored. The result says that if $\mathrm{e}^0_{\mathfrak{m}}(A) \geq 3$, then for every parameter ideal Q in A, one has the equality $I^3 = QI^2$ and that the rings $\mathrm{G}(I)$ and $\mathrm{F}(I)$ are Cohen-Macaulay. This is false unless $\mathrm{e}^0_{\mathfrak{m}}(A) \geq 3$, even though A is a Gorenstein local ring.

Lecture 5: In certain cases, the equality $I^2 = QI$ remains true, unless A is a Cohen-Macaulay local ring. I shall discuss the case where A is a Buchsbaum local ring and also the case where A has FLC, that is the case where A is a genearlized Cohen-Macaulay local ring.

References.

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2. **Prof Srikanth Iyengar** (*University of Nebraska, Lincoln, USA*)
Title: *Using triangulated categories in commutative algebra*

The role of the derived category of a ring as a convenient arena for homological algebra is by now well-established. However, in commutative algebra much of the work involving derived categories uses no more than the fact that it exists. Inspired in part by developments in algebraic topology and representation theory, recent research has began to exploit its structure as a triangulated category to study properties of modules and complexes over commutative noetherian rings. The goal of this lecture series will be to describe some of these developments.

Lecture 1: The derived category of a ring

This lecture will be an introduction to the derived category. I will describe various constructions of the derived category, and its structure as a triangulated category.

Lecture 2: Thick subcategories of the derived category

This lecture will introduce a notion of building one object from another in a triangulated category. It is based on the idea of a thick subcategory of a triangulated category. I will discuss various examples which attest to its relevance to homological algebra. I will also present a theorem of Hopkins and Neeman that classifies the thick subcategories of the category of perfect complexes over commutative noetherian rings. This result is critical input for the results discussed in the next lecture.

Lecture 3: Invariants over local homomorphisms

This lecture will be based on some of the material in [3]. I will prove ascent and descent results of various homological invariants of modules along local homomorphisms of commutative rings. The framework for most of the arguments presented is a method of devissage, based on the Hopkins-Neeman theorem, to be presented in Lecture 2. I will also discuss some special features of the thick subcategory of the derived category of complete intersection rings.

Lecture 4: Levels in triangulated categories

This lecture will introduce a notion of level, with respect to a given class of objects, in a triangulated category. This notion quantifies the building process introduced in Lecture 2, and provides a uniform method for dealing with disparate invariants of modules and complexes, like projective dimension and Loewy length, on an equal footing. Part of the reason for introducing this notion is that it is better behaved under change of categories.

Lecture 5: Homology of perfect complexes

In this lecture I will discuss results on the Loewy length on homology of perfect complexes, obtained using the techniques outlined in Lecture 5. This will be based on [1]

A first approximation to a reference list:

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3. **Prof R. Miró-Roig** (*University of Barcelona, Spain*)

Title: *Lectures in Determinantal ideals*

Standard determinantal ideals, determinantal ideals and symmetric determinantal ideals have been a central topic in both commutative algebra and algebraic geometry and they also have numerous connections with invariant theory, representation theory and combinatorics. Due to their important role, their study has attracted many researchers and has received considerable attention in the literature. In these series of lectures, I will attempt to address the following 3 problrems:

(1) CI-liaison class and G-liaison class of standard determinantal ideals, determinantal ideals and symmetric determinantal ideals.

(2) The multiplicity conjecture for standard determinantal ideals, determinantal ideals and symmetric determinantal ideals.

(3) Unobstructedness and dimension of families of standard determinantal ideals, determinantal ideals and symmetric determinantal ideals.

4. **Prof N. V. Trung** (*Institute of Mathematics, Hanoi, Vietnam*)
Title: *Vertex cover algebras*

The aim of this lecture is to present the relationship between commutative algebra and combinatorics by the structure of vertex cover algebras.

Let Δ be a simplicial complex on the vertex set $[n] = \{1, ..., n\}$. Let $\mathcal{F}(\Delta)$ denote the set of the facets of Δ. We call an integral vector $\mathbf{c} = (c_1, \dots, c_n) \in \mathbb{N}^n$ a *cover* of order k or a k-cover of Δ if $\sum_{i\in F} c_i \geq k$ for all $F \in \mathcal{F}(\Delta)$. If $\mathbf{c}$ is a $(0, 1)$-vector, then $\mathbf{c}$ may be identified with the subset $C = \{i \in [n] : \ c_i \neq 0\}$ of $[n]$. It is clear that $\mathbf{c}$ is a 1-cover if and only if C is a *vertex cover* of Δ in the classical sense, that is, $C \cap F \neq \emptyset$ for all $F \in \mathcal{F}(\Delta)$.

Let $S = K[x_1, \dots, x_n]$ be a polynomial ring in n variables over a field K. Let $A_k(\Delta)$ denote the K-vector space generated by all monomials $x_1^{c_1} \cdots x_n^{c_n} t^k$ such that $(c_1, \dots, c_n) \in \mathbb{N}^n$ is a k-cover of Δ, where t is a new variable. Then

$$A(\Delta) := \bigoplus_{k\geq 0} A_k(\Delta),$$

is a graded S-algebra. We call $A(\Delta)$ the *vertex cover algebra* of Δ.

One may view $A(\Delta)$ as the symbolic Rees algebra of the ideal

$$I^*(\Delta) := \bigcap_{F\in\mathcal{F}(\Delta)} P_F,$$

where P_F denotes the ideal of S generated by the variables x_i with $i \in F$. On the other hand, one may also view $A(\Delta)$ as the toric ring of the affine semigroup generated by the vectors $(\mathbf{c}, k) \in \mathbb{N}^{n+1}$ such that $\mathbf{c}$ is a k-cover of Δ. These interpretations of vertex cover algebras allow us to use different methods of combinatorics and commutative algebra for the study of their structure.

Notice that ideals of the form $I^*(\Delta)$ are exactly squarefree monomial ideals. A counterpart to $I^*(\Delta)$ is the facet ideal $I(\Delta)$ generated by the monomials $x_{i_1} \cdots x_{i_r}$ such that $\{i_1, ..., i_r\}$ is a facet of Δ. If we denote by Δ^* the simplicial complex whose facets are the monomials

$x_{i_1} \cdots x_{i_r}$ such that $\{i_1, ..., i_r\}$ is a minimal vertex cover of Δ, then $I(\Delta) = I^*(\Delta^*)$. Another counterpart of $I^*(\Delta)$ is the Stanley-Reisner ideal I_Δ generated by the monomials $x_{i_1} \cdots x_{i_r}$ such that $\{i_1, ..., i_r\}$ is a non-face of Δ. Thus, one can use vertex cover algebras to study blowup algebras of these squarefree monomial ideals by means of the given simplicial complexes.

In particular, standard graded vertex cover algebras are closely related to a range of hypergraphs which generalize bipartite graphs and trees. These relationships have led to very general results which cover previous major results on blowup algebras of squarefree monomial ideals.